Eur. Phys. J. D **3**, 113–118 (1998)

THE EUROPEAN PHYSICAL JOURNAL D

Cooling mechanisms in potassium magneto–optical traps

C. Fort[1,a], A. Bambini[2], L. Cacciapuoti[1], F.S. Cataliotti[1], M. Prevedelli[1], G.M. Tino[1,b], and M. Inguscio[1]

[1] INFM – Dipartimento di Fisica and European Laboratory for Nonlinear Spectroscopy (L.E.N.S.), Università di Firenze Largo E. Fermi, 2, 50125 Firenze, Italy

[2] Istituto di Elettronica Quantistica, Consiglio Nazionale delle Ricerche via Panciatichi 56/30, 50127 Firenze, Italy

Received: 6 March 1998 / Received in final form: 13 May 1998 / Accepted: 13 May 1998

Abstract. We report on a theoretical and experimental investigation of ^{39}K magneto–optical trapping. The small hyperfine splitting characterizing the upper level of the cooling transition affects the cooling mechanism. In order to model the atom–laser interaction, the whole level structure of the D_2 line has to be taken into account. Two different regimes have been recognized, one optimizing the loading of the trap, the second minimizing the temperature of the atoms. We investigated these two regimes experimentally and found results in agreement with the theoretical predictions.

PACS. 32.80.Pj Optical cooling of atoms; trapping – 42.50.Vk Mechanical effects of light on atoms, molecules, electrons, and ions

1 Introduction

In the list of atoms that were magneto–optically cooled and trapped, potassium has relatively recently been included [1–5]. The peculiarity of potassium magneto–optical traps (MOT) stems from the small hyperfine splitting of the $4p\ ^2P_{3/2}$ excited state [6]. The values of laser detunings used to trap other atoms produce strong hyperfine optical pumping. To solve this problem, in the experiments performed so far on ^{39}K and ^{41}K [1–3], two sets of laser beams were used, one to excite the atoms from the lower $F = 1$ hyperfine level, the other interacting with the atoms in the $F = 2$ level. The frequency of both beams was detuned to the red by an amount larger than the complete hyperfine structure of the excited state. A relatively high laser power was then needed for trapping. In this paper we report on a systematic study of a ^{39}K MOT and demonstrate that the optimum condition for cooling corresponds to a small laser detuning and low intensity. Experimental results agree qualitatively with the prediction of a theoretical model taking into account the complete level structure for the D_2 transition.

2 Theoretical model for cooling in potassium

Due to the particular level structure, the optical trapping and cooling of potassium has peculiar features with respect to other alkalis. In Figure 1 we report the level scheme for the D_2 transition of ^{39}K. The small hyperfine splitting of the involved levels can lead to efficient optical pumping which deeply affects the cooling process. The splitting of the entire excited state ($4p\ ^2P_{3/2}$) manifold is only 5.4 natural line widths ($\Gamma/2\pi = 6.2$ MHz). Also the ground state is split into two closely spaced levels (462 MHz). Because of these facts, the radiative force exerted by the laser fields onto the potassium atoms can depart noticeably from the one evaluated for an isolated transition: in the present case, the laser field can excite Zeeman sublevels belonging to different levels in the multiplet, thus inducing a coherence among them. As a result, transitions are affected by interference processes, and can no longer be regarded as "isolated". Radiative cooling must be evaluated from the relevant matrix elements of the density operator, which in turn are the solution of the Optical Bloch Equations (OBEs) for the whole D_2 manifold.

The construction of the OBEs for potassium is eased, however, by the small level spacing itself. One can assume that non–secular terms in the master equation, arising from the decay of coherences among excited sublevels, do not vanish. Thus, we derive a full set of OBEs by transforming from the eigenstates of the uncoupled Hamiltonian, with angular momenta labeled by $(L, M_L; S, M_S; I, M_I)$, to a fully coupled, hyperfine basis labeled by (F, M_F). Full details can be found in [7]. In what follows we will label by "g" and "e" the levels and sublevels belonging to the ground and excited state, respectively.

In the following, we divide the discussion in two cases: In the first case, we consider a monochromatic field, while in the second case two fields, with a frequency difference

[a] e-mail: fort@colonnello.lens.unifi.it

[b] *Present address:* Dipartimento di Scienze Fisiche, Università di Napoli "Federico II", Complesso Universitario di Monte S. Angelo, via Cintia, 80126 Napoli, Italy

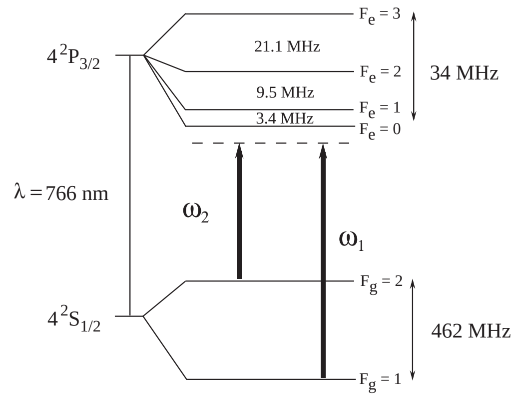

Fig. 1. Hyperfine level structure of the D_2 transition in ^{39}K (not to scale). The laser frequencies ω_1, ω_2 used for magneto–optical trapping this atoms are shown.

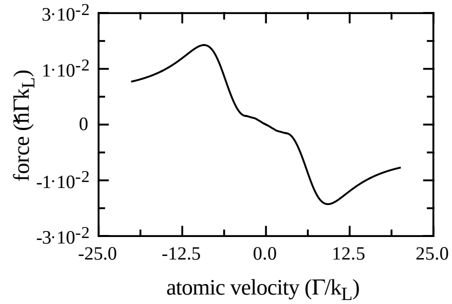

Fig. 2. Radiative force exerted in a $1-D$ configuration in the high intensity regime by the monochromatic field as a function of the atomic velocity. The reduced Rabi frequency is Ω_r= 5.5 Γ and the detuning of the laser with respect to the $F_g = 2 \to F_e = 3$ transition is $-6.5\ \Gamma$.

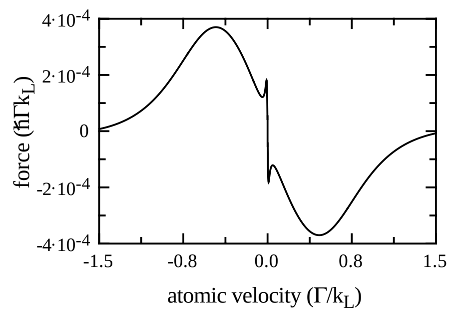

Fig. 3. Radiative force exerted in a $1-D$ configuration in the low intensity regime, by the monochromatic field as a function of the atomic velocity. The reduced Rabi frequency is Ω_r= 0.25 Γ and the detuning of the laser with respect to the $F_g = 2 \to F_e = 3$ transition is $-0.5\ \Gamma$.

close to the ground state hyperfine structure, are considered. Different methods of evaluation are employed in the two cases. Calculations are performed only for the $1-D$ cooling process, in which the atoms move only along the axis of field propagation. No attempt has been made to include the magnetic fields of the MOT.

For the monochromatic case, evaluation of the radiative force was carried out with counter propagating laser fields with opposite circular polarization (σ^+ for the field propagating in the $z+$ direction, and σ^- for the field propagating in the $z-$ direction). In this case, the OBEs can be cast into an autonomous system of linear differential equations by using the Rotating Wave Approximation, and by means of a proper transformation of the density operator, that eliminates the slow time dependence due to the difference of the Doppler–shifted frequencies resulting from the atomic motion. The latter transformation, that is the mathematical counterpart of Larmor's theorem, is performed by passing to a new reference frame that rotates about the z-axis as the atom moves along it [8]. The solution for the resulting system of equations yields the populations and the coherences induced by the field. When the latter are known, the force can be evaluated as in [7]. The advantage of this procedure is that one can also evaluate the force in the high intensity regime. There is no possibility, however, to evaluate separately the dissipative and the reactive components of the force.

In Figure 2 we show the force exerted by the monochromatic field on the atom in the stationary regime as a function of the atomic velocity. The graph corresponds to the case in which the amplitude of the two counter propagating fields is well above the saturation regime (the reduced Rabi frequency $\Omega_r = (\mu E)/\hbar$, where E is the amplitude of the field in one of the polarizations and μ is the reduced matrix element for the $J_g = 1/2 \to J_e = 3/2$, is 5.5 times the value of the spontaneous decay rate Γ), and the field frequency is detuned towards the red side of the transition $F_g = 2 \to F_e = 3$ by 6.5 Γ. The laser frequency is therefore detuned to the red side with respect to the whole manifold of excited–state levels. As is apparent from Figure 2, the main advantage of using such a configuration is that the velocity capture range v_c is large. In these conditions v_c is of the order of 10 Γ/k_L. On the other hand, the friction force exerted by the field is small, and no sub–Doppler cooling feature appears in it. An estimate of the minimum temperature that can be reached in this case is well above the milliKelvin range.

Figure 3 shows the cooling force as a function of the atomic velocity when a small detuning and a low intensity laser field is used instead. In this regime the laser doesn't affect the level linewidth and the problem approaches the general case where the levels of the hyperfine manifold are sufficiently far apart (no power broadening) so that we approach a situation close to that of an isolated transition. The frequency detuning is now $-0.5\ \Gamma$ with respect to the $F_g = 2 \to F_e = 3$ transition, *i.e.*, the laser frequency is tuned just below the frequency of the transition that provides the strongest radiative force, and above all the other transition frequencies of the manifold. In this case, a sub–Doppler, enhanced friction force appears, but the velocity capture range is greatly reduced and the force itself, at the atomic velocity $v \sim 0.5\ \Gamma/k_L$ where it gets the largest value, is still very small. The magnitude of the force is reduced because the low intensity field is unable to populate the $F_g = 2$ state; however a small fraction of the atomic population gets optically pumped continuously into the ground $F_g = 2, M_F = +2$ (for $v < 0$) or into the ground $F_g = 2, M_F = -2$ (for $v > 0$) state. This accounts for the appearance of the sub–Doppler feature. In either case, transitions induced by the σ^+ (σ^-) polarized component of the impinging field dominate over the other σ^- (σ^+) component, and an effective friction force arises.

It is worth mentioning that one could use a monochromatic field tuned near and below the $F_g = 1 \rightarrow F_e = 2$ transition. In this case the optical pumping would accumulate atoms in the $F_g = 2$ ground state. Moreover, the cooling force for the $F_g = 1 \rightarrow F_e = 2$ transition would be smaller because of the lower Zeeman degeneracy of the levels involved.

We consider then the case of a bichromatic field. To counteract the depletion of the $F_g = 2$ ground level due to the optical pumping, a second laser field at a different frequency ω_1 is used, that repumps the atoms into the level that provides the largest friction force (the ground $F_g = 2$ in the case of ^{39}K). The additional field consists of two counter–propagating running waves of opposite circular polarizations, whose frequency is increased by the hyperfine splitting of the ground state with respect to the ω_2 field. It is clear that it is difficult to disentangle the role of the two laser frequencies as a repumper or a cooler, since even the added field can act as a cooling field on the transition $F_g = 1 \rightarrow F_e = 2$, as mentioned above.

A noticeable feature of the two–frequency cooling is the appearance of a beat note in the force exerted by the laser fields onto the atom. There is no stationary state at all, as in the monochromatic cooling, and the coherences as well as the populations oscillate, at the end of the transient regime, at the frequency difference $\omega_1 - \omega_2$. However, because of the large mass of potassium, the effects of the beat note of the force on the atomic motion is negligible, except for an added term in the fluctuations when the atom is at rest, *i.e.*, when the expectation value of the kinetic momentum $\langle p \rangle$ is zero. As a consequence of this phenomenon, however, the force exerted onto the atom cannot be found as in the previous case. To evaluate the optical coherences and hence the total force, one must integrate the full set of OBEs starting from an arbitrary initial condition.

The force in the bichromatic case with intense laser fields at frequencies ω_1 and ω_2, detuned to the red with respect to all the levels in the excited hyperfine manifold (see Fig. 1), shows no sub–Doppler structure. However, a two order of magnitude increase in the force results with respect to the monochromatic case.

An efficient cooling force, when bichromatic irradiation is used, is found in the following configuration: a low intensity ($\Omega_r = 0.5\ \Gamma$) laser tuned below the $F_g = 2 \rightarrow F_e = 3$ transition by 0.5 Γ, and the other laser detuned by the same amount with respect to the $F_g = 1 \rightarrow F_e = 2$ transition. The intensity and frequency of the field at frequency ω_2 are actually the same as those given for the case of Figure 3. In this configuration, the cooling force is not very sensitive to the intensity of the added field at frequency ω_1, but slightly better results were obtained for intensities

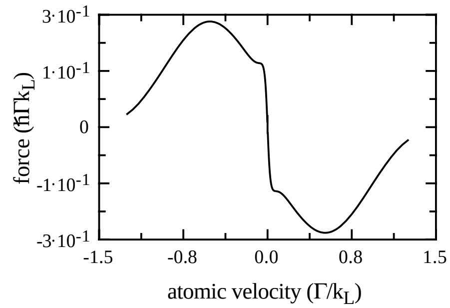

Fig. 4. Friction force as a function of the atomic velocity for bichromatic excitation in $1-D$ configuration in the low intensity regime. The reduced Rabi frequency for the field at frequency ω_2 is Ω_r= 0.55 Γ and its detuning with respect to the $F_g = 2 \rightarrow F_e = 3$ transition is $-0.5\ \Gamma$, while the reduced Rabi frequency for the second field at frequency ω_1 is $\Omega_r = 0.7\ \Gamma$ and its detuning with respect to the $F_g = 1 \rightarrow F_e = 2$ transition is $-0.5\ \Gamma$.

similar to or lower than that of the field at frequency ω_2. Also, low intensity of fields at both frequencies reduce the beat note in the force mentioned above and contribute to a reduction in the fluctuations in the final stage of the cooling process.

The graph in Figure 4 shows the friction force as a function of the atomic velocity in the bichromatic case. The force is much larger than the one shown in Figure 3, and displays a remarkable sub–Doppler structure. An estimate of the $1-D$ minimum temperature reachable in this case gives a value of 1 μK, comparable to the recoil limit of 0.4 μK, and well below the Doppler limit of 150 μK. The temperature limit obtained by taking into account the laser field and vacuum fluctuations at $v \sim 0$, is increased by the fluctuations (of a classical nature) $\langle p^2 \rangle / 2M$, where $\langle p^2 \rangle$ is the time average of the square of the kinetic momentum created by the field fluctuations at the frequency $\omega_1 - \omega_2$.

The reason for such low temperatures is the effectiveness of the sub–Doppler cooling mechanism due to the motion induced orientation [8] in this regime. In addition the induced orientation is enhanced, in our case, since all Zeeman sublevels but the $F_g = 2, M_F = \pm 2$ can be depleted by hyperfine optical pumping in the $F_g = 1$ ground state. This is shown in Figure 5, where the atomic velocity was kept constant at $v = 0.05\ \Gamma/k_L$. Starting from an initial condition in which the $F_g = 1$ sublevels are equally populated, the population of the $F_g = 2$ sublevels evolve with time reaching a situation in which the $M_F = -2$ is highly populated, while the others are almost depleted. The pump field has the only role to counteract the leakage of population into the $F_g = 1$ level.

The limit temperature obtained in this unidimensional configuration cannot be maintained, however, in a three dimensional cooling scheme, since the population accumulated in the highest $|M_F|$ sublevels of the $F_g = 2 \rightarrow F_e = 3$ transition is easily depleted by the transverse laser beams when the atom has a non zero velocity component in the

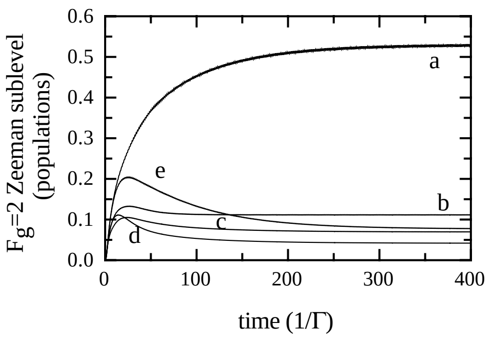

Fig. 5. Time evolution of the population in the Zeeman sublevel of the $F_g = 2$ groundstate, starting from an initial condition where the population is equally distributed in the $F_g = 1$ Zeeman sublevels. The atomic velocity was kept constant at $v = 0.05\ \Gamma/k_L$, the fields parameters are the same as Figure 4. Respectively a, b, c, d, e correspond to the populations of the $F_g = 2$ Zeeman sublevels with $M_F = -2$, $M_F = -1$, $M_F = 0$, $M_F = 1$ and $M_F = 2$.

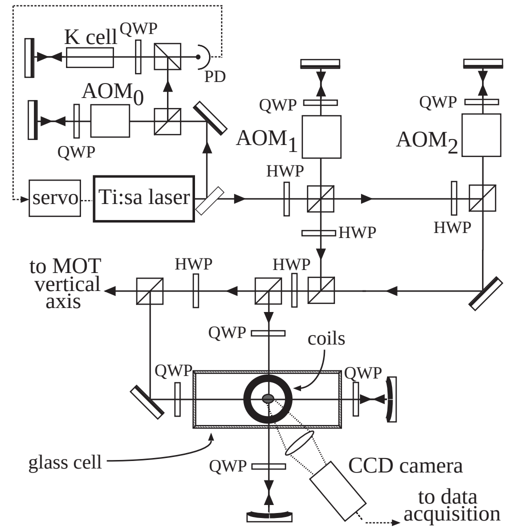

Fig. 6. Scheme of the experimental apparatus. QWP: quarter wave plate; HWP: half wave plate; AOM: acousto–optical modulator; PD: photodiode. Lenses are not shown for clarity.

plane xy. An estimate of the minimum temperature for the three dimensional case gives a value of 20 μK.

The stability of the laser intensity and frequency plays a major role here. Calculations show a severe reduction in the friction force when the field frequency ω_2 is made to fluctuate. This has been evaluated by replacing the frequency detuning δ_2 of the laser frequency from the atomic transition by a function of time $\delta_2(t)$, whose time averaged value $\sqrt{\langle(\delta_2 - \overline{\delta_2})^2\rangle}$ gives an estimate of the fluctuation strength. At $\sqrt{\langle(\delta_2 - \overline{\delta_2})^2\rangle} \sim 3\Gamma$, the friction force is vanishingly small near $v \sim 0$.

A remarkable difference in the latter configuration is that the field at frequency ω_1 must have the same polarization as the one at frequency ω_2. More precisely, the two running waves at different frequency, that propagate in the $z+$ direction, must have the same circular polarization, say σ^+, while the other two beams (propagating in the $z-$ direction) are σ^- polarized. This is not strictly necessary in the small detuning, low intensity regime.

This appears to be a consequence of the fact that, at low intensities, the hyperfine levels other than the ones involved in the cooling transition are poorly populated, and the transition itself retains much of the character of an "isolated transition". At low intensities, the choice of the polarization does not affect the population distribution in the steady state regime. This is not the case for high intensity cooling with large detunings. Reversing the polarization of the repumper beam reduces the population difference of the Zeeman sublevels, which results in a smaller dissipative component of the radiative force. Calculations show that the total force is reduced even more, thus indicating that, in the strong field regime, the Zeeman coherences responsible for the reactive component of the force are highly affected by the choice of the polarizations.

3 Experimental apparatus

In Figure 6 we present a simplified scheme of the experimental apparatus for the ^{39}K MOT. The vacuum system consists of a glass cell connected with a valve to an ion pump that produces a pressure of the order of 10^{-9} Torr. A second valve connects the cell with the potassium reservoir whose temperature can be varied.

All the laser light used in the experiment is provided by a commercial CW Ti:sapphire laser (Coherent model 899-21) with an output power up to 1 W. Using an acousto optic modulator (AOM$_0$) in double–pass configuration, the Ti:sapphire laser frequency is offset locked to the $F_g = 2 \rightarrow F_e = 3$ saturated absorption signal obtained in a potassium vapour cell. Because of the small hyperfine structure of the excited level, we used a configuration with the same circular polarization for both the pump and the probe beams. This strongly reduces the signal from the $F_g = 2 \rightarrow F_e = 1, 2$ transitions. The residual line asymmetry arising from these two transitions gives an uncertainty in the value of laser frequency of about 5 MHz. The two frequencies ω_1 and ω_2, necessary for cooling and trapping potassium atoms, are provided by a couple of AOMs in double–pass configuration. The double–pass configuration allows one to vary the frequency and the intensities of the two laser fields without affecting the alignment of the MOT.

After the beam–shaping optics and AOMs, the maximum power to the trap is ~ 300 mW. The diameter of the beams in the cell is 1.5 cm. The retroreflected beams in the three trapping axes are obtained using curved mirrors in order to compensate for intensity unbalance due to the uncoated cell windows.

The trapping magnetic field has a gradient of 15 G/cm. A photodiode and a CCD camera are used to measure the number of atoms and the size of the cloud in the trap.

4 Results

As discussed above, the cooling mechanisms for potassium are rather peculiar. The discussion in Section 2 suggests that two different regimes must be considered. In the first, obtained with two laser frequencies both detuned to the red with respect to the whole excited–state hyperfine structure, a particularly wide capture range can be achieved. This can allow efficient loading of the MOT from the vapour. In the second scheme discussed above, with smaller detuning and lower intensity of the laser beams, lower temperatures are expected although with a reduced capture range. The combination of the two schemes in succession can then allow trapping of a large number of atoms at temperatures comparable to or even lower than the Doppler cooling limit. In the following, we describe the behaviour of the MOT for these two schemes with particular emphasis on the number of trapped atoms and their temperature.

As already examined in Section 2, in the first configuration both the frequencies give rise to a radiative force. We observed indeed a critical dependence of the MOT operation on the overlap of the two sets of beams, much more critical than for other alkalis, as one would expect in the case in which both laser frequencies contribute to the trapping force. In this case the two trapping regions must coincide. We also studied different beam configurations and the dependence on the beams polarization. Although the maximum number of atoms was obtained with three sets of laser beams all containing both frequencies, we observed trapped atoms also sending only ω_2 light in the x and y directions and only ω_1 in the z direction. In the high–intensity, large–detuning regime, we found a strong dependence on the laser beams polarization. When the polarization of the ω_1 field was changed, optimum operation of the MOT was obtained with the same circular polarization of the two beams, the one determined by the gradient of the magnetic field. As is well known, this is not the case in MOTs of other alkalis where the polarization of the "repumper" laser is not a critical parameter. In our experiment, we recovered this behaviour in the second scheme where the ω_1 laser acts as a "repumper" and does not contribute to the actual force on the atoms.

To evaluate the loading efficiency of the MOT, we measured the number of trapped atoms. The number of atoms collected in the MOT was measured by monitoring the fluorescence in the trapping zone. In Figures 7, 8 and 9 we report the measured number of atoms trapped in the MOT as a function of the intensity of the two fields, and varying the detuning of the ω_2 field with respect to the $F_g = 2 \rightarrow F_e = 3$ transition. The theoretical discussion predicted an efficient loading of the trap for high intensity fields both red detuned with respect to the whole hyperfine structure of the excited level. As expected, the number of atoms trapped in the MOT increases by increasing the intensity of both the laser fields at frequency ω_1 and ω_2 (see Figs. 7, 8). Varying the detuning of the field at frequency ω_2, the total number of atoms loaded in the MOT is maximized with a detuning of about -5Γ with respect to the $F_g = 2 \rightarrow F_e = 3$ transition (Fig. 9).

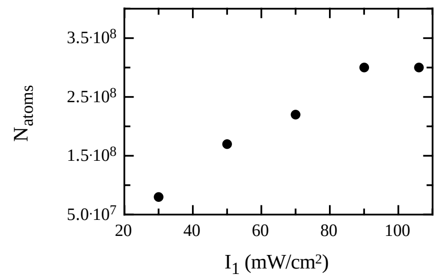

Fig. 7. Number of atoms measured in the MOT as a function of the total intensity I_1 of the laser beam at frequency ω_1. Data are taken with an intensity $I_2 = 190$ mW/cm^2 of the beam at frequency ω_2. Detuning of the two fields are $\Delta_1 = -3\Gamma$ and $\Delta_2 = -5\Gamma$ with respect to the $F_g = 1 \rightarrow F_e = 2$ and $F_g = 2 \rightarrow F_e = 3$ transitions respectively.

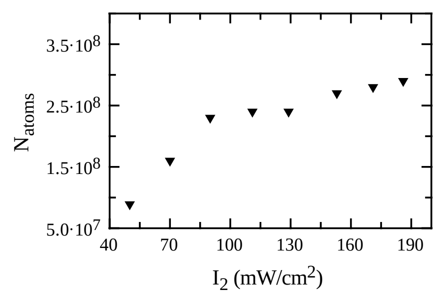

Fig. 8. Number of atoms measured in the MOT as a function of the total intensity I_2 of the laser beam at frequency ω_2. Data are taken with an intensity $I_1 = 110$ mW/cm^2 of the beam at frequency ω_1. Detuning of the two fields are $\Delta_1 = -3\Gamma$ and $\Delta_2 = -5\Gamma$.

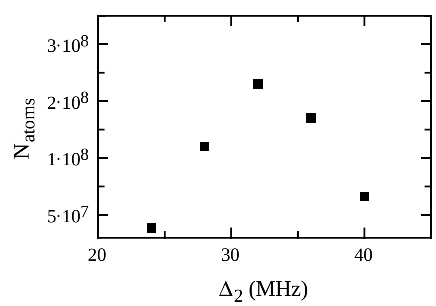

Fig. 9. Number of atoms measured in the MOT as a function of the negative detuning Δ_2 of ω_2 field with respect to the $F_g = 2 \rightarrow F_e = 3$ transition, with $I_1 = 110$ mW/cm^2, $I_2 = 140$ mW/cm^2 and $\Delta_1 = -3\Gamma$.

By using laser parameters which maximize the loading of the MOT, we expect a relatively high temperature as a consequence of the weak slope of the radiative force calculated in this situation. The slope of the radiative force increases with decreasing laser field intensity and detuning. To verify this prediction we studied the temperature of the MOT as a function of the trapping fields parameters. After the loading of the MOT for a few seconds, using laser parameters which optimize the MOT loading, we changed

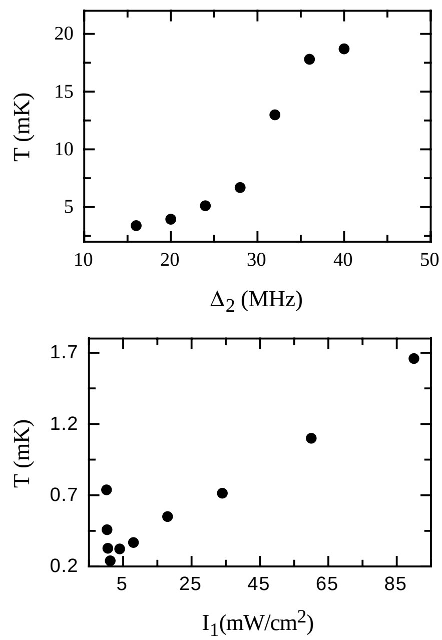

Fig. 10. Temperature of the MOT measured as a function of the laser field parameters: Δ_2 is the negative detuning of the ω_2 field respect to the $F_g = 2 \to F_e = 3$ transition, I_1 is the total intensity of the ω_1 field.

the trapping field parameters for a few ms. The temperature was measured using the release and recapture (R&R) technique [9]. With the intensities and frequencies of the cooling beams that provide the maximum loading of the trap we observe a temperature of a few mK. Changing the parameters of the cooling beams after the loading we reduced the temperature of the trapped atoms by a factor ten. Temperatures measured varying the frequency of the ω_2 field and the intensity of the ω_1 field are presented in Figure 10.

The observed temperature reduction, both decreasing the frequency of ω_2 field and the intensity of ω_1 field, confirms the theoretical predictions. When the intensity of the ω_1 field is reduced below a critical value, the optical pumping is no longer efficient and the temperature increases. The minimum temperature we measure is 240 ± 100 μK. The interpretation of data taken with the R&R method to extract temperature information is very dependent on the modelling of the MOT (atomic density distribution, form of the capture volume, *etc.*). This is why this method can only provide an estimate of the temperature. The uncertainty of temperature measurements makes difficult to determine if we are still above the Doppler limit (150 μK). Furthermore, as discussed in the theoretical section, the limit temperature extracted from the model is strongly affected by the stabilization of the laser intensities and frequencies. The use of a commercial Ti:sapphire laser as source for the cooling radiation poses severe technical restrictions to these points. Our laser system has presently a linewidth of about 2 MHz ($\Gamma/3$) and the uncertainty in the lock point, as already mentioned, is about 1Γ. Moreover, the amplitude noise is around 10% on the time scale of few ms. With this constraint on the laser source we can not reach the conditions suggested by the theory to obtain very cold temperatures.

5 Conclusions

In this paper we presented a study on a ^{39}K MOT. The small hyperfine structure of this atom, strongly influences the cooling mechanism. A theoretical model taking into account the whole hyperfine structure of the D_2 transition was developed. The major results are the necessity of high power fields detuned with respect to the whole hyperfine structure of the excited level, to maximize the loading of the MOT. In this regime a wide velocity capture range is realized, while the high intensity strongly limits the minimum temperature. The theoretical model instead predicts a very low temperature (sub–Doppler) when intensity and frequency of the trapping laser fields are decreased. Indeed, the measured temperature qualitatively follows the prediction, showing a reduction of one order of magnitude in the temperature of the MOT when the intensity of the ω_1 field and the frequency of the ω_2 field are decreased. Unfortunately, a more stringent test of the theory in this regime is presently limited by our laser system. The observed temperature reduction is however already of particular interest in view of BEC experiments in potassium, where both the number of trapped atoms and their temperature must be optimized for an efficient loading of a magnetic trap.

We are grateful to E.A. Cornell and J.R. Ensher for helpful discussion. This work was supported by European Community Council (ECC) under Contract No. ERBFMGECT950017, work partially supported by CNR "Progetto integrato" Contract No. 96.00068.02.

References

1. R.S. Williamson III, T. Walker, J. Opt. Soc. Am. B **12**, 1393 (1995).
2. M.S. Santos, P. Nussenzveig, L.G. Marcassa, K. Helmerson, J. Flemming, S.C. Zilio, V.S. Bagnato, Phys. Rev. A **52**, R4340 (1995).
3. H. Wang, P.L. Gould, W.C. Stwalley, Phys. Rev. A **53**, R1216 (1996).
4. J.A. Behr, A. Gorelov, T. Swanson, O. Häusser, K.P. Jackson, M. Trinczek, U. Giesen, J.M. D'Auria, R. Hardy, T. Wilson, P. Choboter, F. Leblond, L. Buchmann, M. Dombsky, C.D.P. Levy, G. Roy, B.A. Brown, J. Dilling, Phys. Rev. Lett. **79**, 375 (1997).
5. F.S. Cataliotti, E.A. Cornell, C. Fort, M. Inguscio, F. Marin, M. Prevedelli, L. Ricci, G.M. Tino, Phys. Rev. A **57**, 1136 (1998).
6. E. Arimondo, M. Inguscio, P. Violino, Rev. Mod. Phys. **49**, 31 (1977).
7. A. Bambini, A. Agresti, Phys. Rev. A **56**, 3040 (1997).
8. J. Dalibard, C. Cohen–Tannoudji, J. Opt. Soc. Am. B **6**, 2023 (1989).
9. S. Chu, L. Hollberg, J.E. Bjorkholm, A. Cable, A. Ashkin, Phys. Rev. Lett. **55**, 48 (1985).